Note to the problem of asymptotic behavior of viscous incompressible flow around a rotating body

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Abstract

We consider weak solutions to the stationary Navier-Stokes system with Oseen and rotational terms, in an exterior domain. We are interested in the leading term for the velocity field and its gradient. Moreover, we deal with the asymptotic behavior at infinity. We proved the velocity may be split into a constant times the first column of the fundamental solution of the Oseen system, plus a remainder term decaying pointwise near infinity at a rate which is higher than the decay rate of the Oseen tensor. This result improves the theory by M. Kyed, Asymptotic profile of a linearized flow past a rotating body, Q. Appl. Math. 71 (2013), 489-500.

Résumé

Nous considérons des solutions faibles du système de Navier-Stokes stationnaire avec un terme d'Oseen et des termes rotationnels, dans un domaine extérieur. Notre intérêt se porte sur la partie principale d'un développement asymptotique de la vitesse et de son gradient. Nous montrons que la vitesse peut être scindée, à des constantes près, en la première colonne de la solution fondamentale du système d'Oseen ("tenseur d'Oseen"), plus un reste qui décroît ponctuellement dans un voisinage d'infini, à un taux qui est plus élevé que le taux de décroissance du tenseur d'Oseen. Ce résultat améliore la théorie présentée par M. Kyed, Asymptotic profile of a linearized flow past a rotating body, Q. Appl. Math. 71 (2013), 489-500.

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1. Introduction

We shall consider the following problem. Let $\mathfrak{D} \subset \mathbb{R}^3$ be an open bounded set. Suppose this set describes a rigid body moving with constant nonzero translational and angular velocity in an incompressible viscous fluid. Then the flow around this body with respect to a frame attached to this body is governed by the following set of non-dimensional equations (see [7]),

$$-\Delta u + \tau\,\partial_1 u + \tau\,(u \cdot \nabla)u - (\omega \times x) \cdot \nabla u + \omega \times u + \nabla \pi = f, \ \ \operatorname{div} u = 0, \tag{1}$$

in the exterior domain $\overline{\mathfrak{D}}^c := \mathbb{R}^3 \backslash \overline{\mathfrak{D}}$, supplemented by a decay condition at infinity,

$$u(x) \to 0 \ \ \text{for} \ \ |x| \to \infty, \tag{2}$$

and suitable boundary conditions on $\partial \mathfrak{D}$.

In (1) and (2), the functions $u : \overline{\mathfrak{D}}^c \mapsto \mathbb{R}^3$ and $\pi : \overline{\mathfrak{D}}^c \mapsto \mathbb{R}$ are the unknown relative velocity and pressure field of the fluid, respectively, whereas the function $f : \overline{\mathfrak{D}}^c \mapsto \mathbb{R}^3$ stands for a prescribed volume force acting on the fluid. The vector $\tau\,(-1, 0, 0)$ represent the uniform velocity of the flow at infinity or the velocity of the body, depending on the physical situation, and $\omega := \varrho \cdot (1, 0, 0)$ corresponds to the constant angular velocity of the body. In particular the vectors of translational and angular velocity are parallel. From a physical point of view this assumption is natural for a steady flow. The parameters $\tau \in (0, \infty)$ and $\varrho \in \mathbb{R} \backslash \{0\}$ are dimensionless quantities that can be identified with the Reynolds and Taylor number, respectively. They will be considered as fixed, like the domain $\mathfrak{D}$.

We are interested in "Leray solutions" of (1), (2), that is, weak solutions characterized by the conditions $u \in L^6(\overline{\mathfrak{D}}^c)^3 \cap W_{loc}^{1,1}(\overline{\mathfrak{D}}^c)^3$, $\nabla u \in L^2(\overline{\mathfrak{D}}^c)^9$ and $\pi \in L_{loc}^2(\overline{\mathfrak{D}}^c)$.

From [8] and [3] it follows that the velocity part u of a Leray solution (u, π) to (1), (2) decays for $|x| \to \infty$ as expressed by the estimates

$$|u(x)| \le C\,\big(|x|\,s(x)\big)^{-1}, \quad |\nabla u(x)| \le C\,\big(|x|\,s(x)\big)^{-3/2} \tag{3}$$

for $x \in \mathbb{R}^3$ with $|x|$ sufficiently large, where $s(x) := 1 + |x| - x_1$ $(x \in \mathbb{R}^3)$ and $C > 0$ a constant independent of x. The factor $s(x)$ may be considered as a mathematical manifestation of the wake extending downstream behind a body moving in a viscous fluid.

By Kyed [9] it was shown that

$$u_j(x) = \gamma\,E_{j1}(x) + R_j(x), \quad \partial_l u_j(x) = \gamma\,\partial_l E_{j1}(x) + S_{jl}(x) \ \ (x \in \overline{\mathfrak{D}}^c,\ 1 \le j, l \le 3), \tag{4}$$

where $E : \mathbb{R}^3 \backslash \{0\} \mapsto \mathbb{R}^4 \times \mathbb{R}^3$ denotes a fundamental solution to the Oseen system

$$-\Delta v + \tau\,\partial_1 v + \nabla \Pi = f, \ \ \operatorname{div} v = 0 \ \ \text{in} \ \ \mathbb{R}^3. \tag{5}$$

The definition of the function E is stated in Section 2. As becomes apparent from this definition, the term $E_{j1}(x)$ may be expressed explicitly in terms of elementary functions. The coefficient γ is also given explicitly, its definition involving the Cauchy stress tensor. The remainder terms R and S are characterized by the relations $R \in L^q(\overline{\mathfrak{D}}^c)^3$ for $q \in (4/3, \infty)$, $S \in L^q(\overline{\mathfrak{D}}^c)^3$ for $q \in (1, \infty)$. Since it is known from [6, Section VII.3] that $E_{j1}|B_r^c \notin L^q(B_r^c)$ for $r > 0$, $q \in [1, 2]$, and $\partial_l E_{j1}|B_r^c \notin L^q(B_r^c)$ for $r > 0$, $q \in [1, 4/3]$, $j, l \in \{1, 2, 3\}$, the function R decays faster than E_{j1}, and S_{jl} faster than $\partial_l E_{j1}$, in the sense of L^q-integrability. Thus the equations in (4) may in fact be considered as asymptotic expansions of u and ∇u, respectively. The theory in [9] is valid under the assumption that u verifies the boundary conditions

$$u(x) = e_1 + (\omega \times x) \quad \text{for} \ \ x \in \partial \mathfrak{D}. \tag{6}$$

Reference [9] does not deal with pointwise decay of R and S, nor does it indicate whether $S = \nabla R$.

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In Theorem 3.1 below we derive a pointwise decay of respectively u and ∇u, which is independent on the boundary conditions but in comparison with [9] and indicated in (4) our leading term is less explicit than the term $\gamma E_{j1}(x)$ in (4) and instead of the fundamental solution $E_{j1}(x)$ of the stationary Oseen system, we use the time integral of the fundamental solution of the evolutionary Oseen system.

In [5] it was shown that $\mathcal{Z}_{j1}(x,0) = E_{j1}(x)$ for $x \in \mathbb{R}^3 \backslash \{0\}$, $1 \leq j \leq 3$, and $\lim_{|x| \to \infty} |\partial_x^\alpha \mathcal{Z}_{jk}(x,0)| = O\big((|x|\,s(x))^{-3/2-|\alpha|/2}\big)$ for $1 \leq j \leq 3$, $k \in \{2,3\}$ ([[5] Corollary 4.5, Theorem 5.1]). Thus, setting

$$\mathfrak{G}_j(x) := \sum_{k=2}^3 \beta_k\, \mathcal{Z}_{jk}(x,0) + \mathfrak{F}_j(x) \quad (x \in \overline{B_{S_1}}^c,\ 1 \leq j \leq 3), \tag{7}$$

we may deduce from (16) that

$$u_j(x) = \beta_1\, E_{j1}(x) + \left(\int_{\partial\Omega} u \cdot n\, do_x\right) x_j\, (4\pi\,|x|^3)^{-1} + \mathfrak{G}_j(x) \quad (x \in \overline{B_{S_1}}^c,\ 1 \leq j \leq 3) \tag{8}$$

and

$$\lim_{|x| \to \infty} |\partial^\alpha \mathfrak{G}(x)| = O\big((|x|\,s(x))^{-3/2-|\alpha|/2}\,\ln(2+|x|)\big) \quad \text{for } \alpha \in \mathbb{N}_0^3 \text{ with } |\alpha| \leq 1 \tag{9}$$

(Theorem 3.2, Corollary 3.3). If we compare how the coefficient γ from (4) is defined in [9], and the coefficient β_1 from (21) in [4] (see Theorem 3.1 below), and if we take account of the boundary condition (6) satisfied by u in [9], we see that γ and β_1 coincide.

2. Notation. Definition of fundamental solutions. Auxiliary results.

By $|\ |$ we denote the Euclidean norm in $\mathbb{R}^3$ and the length $\alpha_1 + \alpha_2 + \alpha_3$ of a multiindex $\alpha \in \mathbb{N}_0^3$. Put $e_1 := (1,0,0)$. For $r > 0$, we set $B_r := \{y \in \mathbb{R}^3 : |y| < r\}$. If $A \subset \mathbb{R}^3$, we put $A^c := \mathbb{R}^3 \backslash A$. Recall the abbreviation $s(x) := 1 + |x| - x_1$ $(x \in \mathbb{R}^3)$ introduced in Section 1.

If $A \subset \mathbb{R}^3$ is open, $p \in [1,\infty)$ and $k \in \mathbb{N}$, we write $W^{k,p}(A)$ for the usual Sobolev space of order k and exponent p. If $B \subset \mathbb{R}^3$ is again an open set, we define $L_{loc}^p(B)$, $W_{loc}^{k,p}(B)$ as the set of all functions $v : B \mapsto \mathbb{R}$ such that $v|_U \in L^p(U)$ and $v|_U \in W^{k,p}(U)$, respectively, for any open bounded set $U \subset \mathbb{R}^3$ with $\overline{U} \subset B$.

For the Fourier transform $\widehat{g}$ of a function $g \in L^1(\mathbb{R}^3)$, we choose the definition $\widehat{g}(\xi) := (2\pi)^{-3/2} \int_{\mathbb{R}^3} e^{-i\xi x} g(x)\, dx$ $(\xi \in \mathbb{R}^3)$. This fixes the definition of the Fourier transform of a tempered distribution as well.

The numbers $\tau \in (0,\infty)$, $\varrho \in \mathbb{R} \backslash \{0\}$ introduced in Section 1 will be kept fixed throughout. Recall that the vector ω is given by $\omega := \varrho \cdot e_1$. We introduce a matrix $\Omega \in \mathbb{R}^{3 \times 3}$ by setting

$$\Omega := \varrho \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}.$$

Note that $\omega \times x = \Omega \cdot x$ for $x \in \mathbb{R}^3$. We write $\mathfrak{C}$ for positive constants that may depend on τ or ϱ. Constants additionally depending on parameters $\sigma_1, ..., \sigma_n \in (0,\infty)$ for some $n \in \mathbb{N}$ are denoted by $\mathfrak{C}(\sigma_1, ..., \sigma_n)$. We state some inequalities involving $s(x)$ or $x - \tau\,t\,e_1$. Next we introduce some fundamental solutions. Put $N(x) := (4\pi\,|x|)^{-1}$ for $x \in \mathbb{R}^3 \backslash \{0\}$ ("Newton potential", fundamental solution of the Poisson equation

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in $\mathbb{R}^3$), $\mathfrak{O}(x) := (4\pi\,|x|)^{-1}\,e^{-\tau\,(|x|-x_1)/2}$ for $x \in \mathbb{R}^3\backslash\{0\}$ (fundamental solution of the scalar Oseen equation $-\Delta v + \tau\,\partial_1 v = g$ in $\mathbb{R}^3$), $K(x,t) := (4\pi\,t)^{-3/2}\,e^{-|x|^2/(4\,t)}$ for $x \in \mathbb{R}^3$, $t \in (0,\infty)$ (fundamental solution of the heat equation in $\mathbb{R}^3$),

$\psi(r) := \int_0^r (1 - e^{-t})\,t^{-1}\,dt$ $(r \in \mathbb{R})$, $\quad \Phi(x) := (4\pi\,\tau)^{-1}\,\psi\big(\tau\,(|x| - x_1)/2\big)$ $(x \in \mathbb{R}^3)$,

$E_{jk}(x) := (\delta_{jk}\,\Delta - \partial_j\partial_k)\Phi(x)$, $\quad E_{4k}(x) := x_k\,(4\pi\,|x|^3)^{-1}$ $(x \in \mathbb{R}^3\backslash\{0\}$, $1 \le j,k \le 3)$ (fundamental solution of the Oseen system (5), with $(E_{jk})_{1\le j,k\le3}$ the velocity part and $(E_{4k})_{1\le k\le3}$ the pressure part). Following Solonnikov [10, (40)]we introduce the velocity part $(T_{jk})_{1\le j,k\le3}$ of a fundamental solution of the time-dependent Stokes system, setting $T_{jk}(x,t) := \delta_{jk}\,K(x,t) + \partial_j\partial_k\big(\int_{\mathbb{R}^3} N(x-y)\,K(y,t)\,dy\big)$ $(x \in \mathbb{R}^3$, $t > 0$, $1 \le j,k \le 3)$.

Put

$$\Gamma(x,y,t) := T(x - \tau\,t\,e_1 - e^{-t\,\Omega}\cdot y,\,t)\cdot e^{-t\,\Omega} \quad \text{for } x,y \in \mathbb{R}^3, \, t > 0. \tag{10}$$

The matrix-valued function $(\Gamma_{jk})_{1\le j,k\le3}$ is the velocity part of a fundamental solution to the time-dependent variant of the linearized equation of (1), precisely

$$-\Delta v + \tau\,\partial_1 v - (\omega \times x)\cdot\nabla v + \omega \times v + \nabla\Pi = f, \quad \mathrm{div}\,v = 0. \tag{11}$$

This fundamental solution was constructed by Guenther, Thomann [11] via a procedure involving Kummer functions, an approach also used in [1] – [5]. However, Guenther, Thomann [11, (3.9)] showed that Γ is given by (10) as well, thus providing an access to this function which is more convenient in many respects.

From [1] we have $\int_0^\infty |\Gamma(x,y,t)|\,dt < \infty$ for $x,y \in \mathbb{R}^3$ with $x \ne y$, so we may define

$$\mathcal{Z}_{jk}(x,y) := \int_0^\infty \Gamma_{jk}(x,y,t)\,dt \quad \text{for } x,y \in \mathbb{R}^3 \text{ with } x \ne y, \, 1 \le j,k \le 3.$$

This function $\mathcal{Z}$ was introduced on [11, p. 96] as the velocity part of a fundamental solution to (11).

3. Statement of our main result.

Theorem 3.1 Let $\mathfrak{D} \subset \mathbb{R}^3$ be open, $p \in (1,\infty)$, $f \in L^p(\mathbb{R}^3)^3$ with $supp(f)$ compact. Let $S_1 \in (0,\infty)$ with $\overline{\mathfrak{D}} \cup supp(f) \subset B_{S_1}$.
Let $u \in L^6(\overline{\mathfrak{D}}^c)^3 \cap W_{loc}^{1,1}(\overline{\mathfrak{D}}^c)^3$, $\pi \in L_{loc}^2(\overline{\mathfrak{D}}^c)$ with $\nabla u \in L^2(\overline{\mathfrak{D}}^c)^9$, $div\,u = 0$ and

$$\int_{\overline{\mathfrak{D}}^c}\Big[\nabla u\cdot\nabla\varphi + \big(\tau\,\partial_1 u + \tau\,(u\cdot\nabla)u - (\omega\times z)\cdot\nabla u + \omega\times u\big)\cdot\varphi - \pi\,div\,\varphi\Big]dz \tag{12}$$

$$= \int_{\overline{\mathfrak{D}}^c} f\cdot\varphi\,dz \quad \text{for } \varphi \in C_0^\infty(\overline{\mathfrak{D}}^c)^3.$$

(This means the pair (u,π) is a Leray solution to (1), (2).) Suppose in addition that

$$\mathfrak{D} \text{ is } C^2\text{-bounded}, \quad u|\partial\mathfrak{D} \in W^{2-1/p,\,p}(\partial\mathfrak{D})^3, \quad \pi|B_{S_1}\backslash\overline{\mathfrak{D}} \in L^p(B_{S_1}\backslash\overline{\mathfrak{D}}). \tag{13}$$

Let n denote the outward unit normal to $\mathfrak{D}$, and define

$$\beta_k := \int_{\overline{\mathfrak{D}}^c} f_k(y)\,dy \tag{14}$$

$$+ \int_{\partial\mathfrak{D}}\sum_{l=1}^3\big(-\partial_l u_k(y) + \delta_{kl}\,\pi(y) + (\tau\,e_1 - \omega\times y)_l\,u_k(y) - \tau\,(u_l\,u_k)(y)\big)\,n_l(y)\,do_y \tag{15}$$

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for $1 \leq k \leq 3$,

$$\mathfrak{F}_j(x) := \int_{\overline{\mathfrak{D}}^c} \Big[\sum_{k=1}^{3} \big(\mathcal{Z}_{jk}(x,y) - \mathcal{Z}_{jk}(x,0) \big) f_k(y) - \tau \cdot \sum_{k,l=1}^{3} \mathcal{Z}_{jk}(x,y) (u_l \, \partial_l u_k)(y) \Big] dy$$

$$+ \int_{\partial\mathfrak{D}} \sum_{k=1}^{3} \Big[\big(\mathcal{Z}_{jk}(x,y) - \mathcal{Z}_{jk}(x,0) \big) \sum_{l=1}^{3} \big(-\partial_l u_k(y) + \delta_{kl} \, \pi(y) + (\tau \, e_1 - \omega \times y)_l \, u_k(y) \big) \, n_l(y)$$

$$+ \big(E_{4j}(x-y) - E_{4j}(x) \big) \, u_k(y) \, n_k(y)$$

$$+ \sum_{l=1}^{3} \big(\partial_{y_l} \mathcal{Z}_{jk}(x,y) \, (u_k \, n_l)(y) + \tau \mathcal{Z}_{jk}(x,0) \, (u_l \, u_k \, n_l)(y) \big) \Big] \, do_y$$

for $x \in \overline{B_{S_1}}^c$, $1 \leq j \leq 3$. The preceding integrals are absolutely convergent. Moreover $\mathfrak{F} \in C^1(\overline{B_{S_1}}^c)^3$ and equation

$$u_j(x) = \sum_{k=1}^{3} \beta_k \, \mathcal{Z}_{jk}(x,0) + \Big(\int_{\partial\Omega} u \cdot n \, do_x \Big) x_j \, (4\pi \, |x|^3)^{-1} + \mathfrak{F}_j(x) \tag{16}$$

holds. In addition, for any $S \in (S_1, \infty)$, there is a constant $C > 0$ which depends on τ, ϱ, S_1, S, f, u and π, and which is such that

$$|\partial^\alpha \mathfrak{F}(x)| \leq C \big(|x| \, s(x) \big)^{-3/2 - |\alpha|/2} \, \ln(2 + |x|) \quad \text{for } x \in \overline{B_S}^c, \ \alpha \in \mathbb{N}_0^3 \text{ with } |\alpha| \leq 1. \tag{17}$$

Proof : We will just give main lines of proof. For complete details see [4, Theorem 3.1]. The main tool of proof is the representation formula (12) with

$$u_j(y) = \mathcal{R}_j(f - \tau(u \cdot \nabla)u)(y) + \mathcal{B}_j(u, \pi(y)), \tag{18}$$

where

$$\mathcal{B}_j(y) := \mathcal{B}_j(u, \pi)(y) \tag{19}$$

$$:= \int_{\partial\mathfrak{D}} \sum_{k=1}^{3} \Big[\sum_{l=1}^{3} \Big(\mathcal{Z}_{jk}(y,z) \cdot \big(-\partial_l u_k(z) + \delta_{kl} \cdot \pi(z) + u_k(z) \cdot (\tau \cdot e_1 - \omega \times z)_l \big)$$

$$+ \partial_{z_l} \mathcal{Z}_{jk}(y,z) \cdot u_k(z) \Big) \cdot n_l^{(\mathfrak{D})}(z) + E_{4j}(y-z) \cdot u_k(z) \cdot n_k^{(\mathfrak{D})}(z) \Big] \, do_z$$

for $y \in \overline{\mathfrak{D}}^c$, with outer normal $n^{(\mathfrak{D})}$ to $\mathfrak{D}$.

We have to consider term $\mathcal{R}_j((u \cdot \nabla)u)(x) = \int_{\overline{\mathcal{D}}^c} \sum_{k=1}^{3} \mathcal{Z}_{jk}(x,y)[(u \cdot \nabla)u_k](y) dy$. Applying the integration by parts, [1, Lemma 4.2, Theorem 2.19, Theorem 5.5, Theorem 3.3] and [2, Theorem 1.1] we get corresponding estimate. Concerning leading term for gradient of velocity $\int_{\overline{\mathcal{D}}^c} \sum_{k=1}^{3} \partial_m \mathcal{Z}_{jk}(x,y)[(u \cdot \nabla)u_k](y) dy$. we have first to divide the integral on two parts $\int_{\overline{\mathcal{D}}^c \setminus B_1(x)}$ and $\int_{B_1(x)}$. Again integrating by parts and using [1, Lemmas 3.3, 5.4], and [2, Theorem 1] we get the leading term for the velocity part.

Theorem 3.2 *Let $\mathfrak{D}$, p, f, S_1, u, π satisfy the assumptions of Theorem 3.1, including (13). Let β_1, β_2, β_3 and $\mathfrak{F}$ be defined as in Theorem 3.1. Define the function $\mathfrak{G}$ as*

$$\mathfrak{G}_j(x) := \sum_{k=2}^{3} \beta_k \, \mathcal{Z}_{jk}(x,0) + \mathfrak{F}_j(x) \quad (x \in \overline{B_{S_1}}^c, \ 1 \leq j \leq 3). \tag{20}$$

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Then $\mathfrak{G} \in C^1(\overline{B_{S_1}}^c)^3$, *equation*

$$u_j(x) = \beta_1\, E_{j1}(x) + \left(\int_{\partial\Omega} u \cdot n\, do_x \right) x_j\, (4\,\pi\,|x|^3)^{-1} + \mathfrak{G}_j(x) \quad (x \in \overline{B_{S_1}}^c,\ 1 \le j \le 3) \tag{21}$$

holds, and for any $S \in (S_1, \infty)$, *there is a constant* $C > 0$ *which depends on* τ, ϱ, S_1, S, f, u *and* π, *and which is such that*

$$|\partial^\alpha \mathfrak{G}(x)| \le C \left(|x|\, s(x) \right)^{-3/2 - |\alpha|/2} \ln(2 + |x|) \quad \text{for } x \in \overline{B_S}^c,\ \alpha \in \mathbb{N}_0^3 \text{ with } |\alpha| \le 1.$$

Corollary 3.3 *Take* $\mathfrak{D}$, p, f, S_1, u, π *as in Theorem 3.1, but without requiring (13). (This means that* (u, π) *is only assumed to be a Leray solution of (1), (2).) Put* $\widetilde{p} := \min\{3/2, p\}$.
 Then $u \in W_{loc}^{2,\widetilde{p}}(\overline{\mathfrak{D}}^c)^3$ *and* $\pi \in W_{loc}^{1,\widetilde{p}}(\overline{\mathfrak{D}}^c)$.
 Fix some number $S_0 \in (0, S_1)$ *with* $\overline{\mathfrak{D}} \cup supp(f) \subset B_{S_0}$, *and define* β_1, β_2, β_3 *and* $\mathfrak{F}$ *as in Theorem 3.1, but with* $\mathfrak{D}$ *replaced by* B_{S_0}, *and* $n(x)$ *by* $S_0^{-1} x$, *for* $x \in \partial B_{S_0}$. *Moreover, define* $\mathfrak{G}$ *as in (20).*
 Then all the conclusions of Theorem 3.2 are valid.

 Proof : Proof is based on the Fourier transformation of the Oseen resolvent to get Fourier transformation of our fundamental solution and applying [11, Lemma 13]. For complete details see [5].

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